Superroughening versus intrinsic anomalous scaling of surfaces

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We study kinetically rough surfaces which display anomalous scaling in local properties such as the roughness or the height-difference correlation function. By studying the power spectrum of the surface and its relation to the height-difference correlation, we distinguish two independent causes for anomalous scaling. One is superroughening (global roughness exponent larger than or equal to 1), even if the spectrum behaves nonanomalously. Another cause is what we term an intrinsically anomalous spectrum, in whose scaling an independent exponent exists, which induces different scaling properties for small and large length scales. We show that in this case the surface does not need to be superrough in order to display anomalous scaling. The scaling relations we propose for the structure factor and height-difference correlation for intrinsically anomalous surfaces are shown to hold for a random diffusion equation, independently of the value of the global roughness exponent below or above one. [S1063-651X(97)06910-9]

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I. INTRODUCTION

In recent years, an enormous amount of work has been devoted to the study of the dynamics of growing surfaces [1–4]. Although in most cases the growth processes considered occur very far from equilibrium, it has been observed that the surface fluctuations exhibit power law behavior similar to that found at second order transitions between equilibrium phases. For instance, if we measure the fluctuations of a $(d+1)$-dimensional interface by the *global* width

$$W(L,t)=\langle\langle[h(\vec{x},t)-\overline{h}(t)]^2\rangle_{\vec{x}}^{1/2}\rangle, \tag{1}$$

where $\langle\ \rangle_{\vec{x}}$ denotes spatial average over the whole substrate of size L^d, $\overline{h}(t)\equiv\langle h(\vec{x},t)\rangle_{\vec{x}}$ is the average value of the height at time t, and $\langle\ \rangle$ is an average over realizations of the noise, in many cases it is observed that $W(L,t)$ satisfies the dynamic scaling *ansatz* of Family and Vicsek [5],

$$W(L,t)=t^{\chi/z}f(L/\xi(t)), \tag{2}$$

where the scaling function $f(u)$ behaves as

$$f(u)\sim\begin{cases}\text{const} & \text{if} \quad u\gg 1\\ u^{\chi} & \text{if} \quad u\ll 1.\end{cases} \tag{3}$$

The roughness exponent χ characterizes the surface morphology in the stationary regime, in which the horizontal correlation length $\xi(t)\sim t^{1/z}$ (z is the so-called dynamic exponent) has reached a value larger than the system size L. This happens for times larger than the saturation time, $t\gg t_s(L)$, which scales with L as $t_s(L)\sim L^z$. The ratio $\beta=\chi/z$ is called the growth exponent and characterizes the short time behavior of the surface.

The existence of dynamic scaling is supposed to arise from the self-affine character of the interface. Scale invariance implies that there is no characteristic length scale in the surface besides the system size, and thus all scales obey the same physics. In particular the *local* width measuring the surface fluctuations over a window of size $\ell\ll L$ should scale in the same way as Eq. (2), hence

$$w(\ell,t)\sim\begin{cases}t^{\beta} & \text{if} \quad t^{1/z}\ll\ell\\ \ell^{\chi} & \text{if} \quad \ell\ll t^{1/z}.\end{cases} \tag{4}$$

Let us stress that this scaling behavior for the *local* width is *not* guaranteed in general when Family-Vicsek scaling holds for the global width (3), since the self-affinity of the interface is an additional independent condition.

Despite the success of the above scaling picture in the characterization of many growth models [1–4], it has been known that in the case of the so-called *superrough* surfaces, i.e., for surfaces with a global roughness exponent $\chi>1$, the usual assumption of the equivalence between the global and local descriptions of the surface is no longer valid [6–8]. In these systems, the usual Family-Vicsek form (4) for the local width has to be replaced by

$$w(\ell,t)\sim\begin{cases}t^{\beta} & \text{if} \quad t^{1/z}\ll\ell\\ \ell^{\chi_{\rm loc}}t^{\beta_*} & \text{if} \quad \ell\ll t^{1/z}\ll L\\ \ell^{\chi_{\rm loc}}L^{z\beta_*} & \text{if} \quad \ell\ll L\ll t^{1/z},\end{cases} \tag{5}$$

where $\chi_{\rm loc}=1$ is the so-called *local* roughness exponent and $\beta_*=(\chi-\chi_{\rm loc})/z$. This behavior has been termed *anomalous scaling* in the literature [7,9] and lately different models have been studied in which $\chi_{\rm loc}$ and β_* take values different from 1 and $\beta-1/z$, respectively [10]. In the presence of anomalous scaling, $\chi_{\rm loc}$ and χ differ, hence not all length scales are equivalent in the system. The common understanding in the literature has been that anomalous scaling is related to the superrough character of the surfaces studied. However, in a recent paper [11] two of us have demonstrated that growth models in which $\chi<1$ may also exhibit an unconventional scaling behavior with scaling relations between exponents similar to those in Eq. (5) [12].

In this paper we complete the picture presented in Ref. [11], showing that the mechanisms which lead to anomalous scaling behavior can be separated into two classes, according to the behavior of the structure factor or power spectrum of the surface, $S(k,t)$, to be defined below. One of the mechanisms is superroughening, which occurs when $S(k,t)$ has the Family-Vicsek form but $\chi>1$. The second independent mechanism corresponds to what we term *intrinsic* anomalous scaling of the structure factor, where an independent exponent appears which measures the difference between the short and large length scale power laws, namely, the difference between the local and global roughness exponents $\chi_{\rm loc}$ and χ. We will show how to identify the scaling by extracting the independent critical exponents from the correlation functions. Intrinsic anomalous scaling of the structure factor has been already measured in some discrete growth models [9] relevant to epitaxial growth of surfaces. However, since all the surfaces studied featured $\chi>1$, a clear distinction had not been made concerning the origin of their anomalous scaling, which had been implicitly associated with superroughness. Here we show that, on the contrary, surfaces with anomalous spectra display anomalous scaling *no matter* what their value of the global roughness exponent is, either $\chi>1$ or $\chi<1$.

The paper is organized as follows. In Sec. II we present the two different shapes of the structure factor which can give rise to anomalous scaling of the local width, namely, a Family-Vicsek structure factor with $\chi>1$ and an intrinsically anomalous power spectrum, and comment on the relation to similar scaling relations already proposed in the literature [9]. In Sec. III we present numerical simulations of an analytically solvable model which features intrinsic anomalous scaling independent of the value of its global roughness exponent χ. We show that the model has an intrinsically anomalous structure factor. Finally, we present some general conclusions in Sec. IV.

II. ANOMALOUS DYNAMIC SCALING

A convenient way of investigating the scaling of a surface is to compute the height-difference correlation function

$$G(\ell,t)=\langle\langle[h(\vec{\ell}+\vec{x},t)-h(\vec{x},t)]^2\rangle_{\vec{x}}\rangle. \tag{6}$$

This correlation function scales in the same way as the square of the local width, $G(\ell,t)\sim w^2(\ell,t)$, and provides an alternative method to determine the critical exponents.

The complete dynamic scaling can also be obtained by studying the Fourier transform of the interface height in a system of linear size L (see, e.g., [1], and references therein), $\hat{h}(\vec{k},t)=L^{-d/2}\Sigma_{\vec{x}}[h(\vec{x},t)-\overline{h(t)}]\exp(i\vec{k}\cdot\vec{x})$. In this representation, the properties of the surface can be investigated by calculating the structure factor or power spectrum, $S(k,t)=\langle\hat{h}(\vec{k},t)\hat{h}(-\vec{k},t)\rangle$, which contains the same information on the system [13] as the height-difference correlation function $G(\ell,t)$ defined in Eq. (6), both of them being related by

$$G(\ell,t)\propto\int\frac{d^d\vec{k}}{(2\pi)^d}[1-\cos(\vec{k}\cdot\vec{\ell})]S(k,t), \tag{7}$$

where the momentum integral is limited to $2\pi/L\le k\le\pi/a$ and represents a continuum approximation to the sum over the discrete set of modes. In a discrete growth model, a is identified with the lattice spacing.

A. Superroughening

When expressed in terms of the structure factor, Family-Vicsek scaling reads

$$S(k,t)=k^{-(2\chi+d)}s_{\rm FV}(kt^{1/z}), \tag{8}$$

with $s_{\rm FV}$ the following scaling function:

$$s_{\rm FV}(u)\sim\begin{cases}\text{const} & \text{if}\quad u\gg1\\ u^{2\chi+d} & \text{if}\quad u\ll1.\end{cases} \tag{9}$$

Indeed, Eqs. (8) and (9) can easily be seen to be equivalent to Eqs. (2) and (3), by noting that the global width is nothing but the integral of $S(k,t)$, i.e.,

$$W^2(L,t)=\frac{1}{L^d}\sum_{\vec{k}}S(k,t)=\int\frac{d^d\vec{k}}{(2\pi)^d}S(k,t). \tag{10}$$

Note as well that Eq. (9) implies that, for $kt^{1/z}\gg1$, the spectrum does *not* depend on time, and hence at saturation $(t^{1/z}\gg L)$, $S(k,t)$ is a pure power law independent of system size. Going back to real space, Eq. (9) implies, using Eq. (7), the usual Family-Vicsek scaling form (4) for $G(\ell,t)$.

In the case of growth models generating superrough surfaces $(\chi>1)$, but with a *structure factor fulfilling Family-Vicsek scaling*, the integrals in Eq. (7) are divergent in the limit $\ell\ll t^{1/z}$ for $L\to\infty$, given the strong singularity at the origin of integration. Taking the limit $\ell\ll t^{1/z}$ first for fixed a, L, one obtains

$$G(\ell,t)\sim\begin{cases}\ell^2t^{2(\chi-1)/z} & \text{if}\quad \ell\ll t^{1/z}\ll L\\ \ell^2L^{2(\chi-1)} & \text{if}\quad \ell\ll L\ll t^{1/z},\end{cases} \tag{11}$$

so that comparing with Eq. (5) $\chi_{\rm loc}=1$ and $\beta_*=\beta-1/z$. In the early time regime $t^{1/z}\ll\ell\ll L$, $G(\ell,t)\sim t^{2\chi/z}$. As has been remarked in Refs. [8,14], the fact that $\chi_{\rm loc}$ cannot exceed 1 for superrough surfaces $(\chi>1)$ is a purely geometric property which follows from definition (6). A very well-known example of the scaling (11) is the so-called linear molecular beam epitaxy equation [6,7].

B. Intrinsic anomalous scaling

There exist growth models —an example of which is discussed in the following section, and see also [9]— for which the *the structure factor* presents an unconventional scaling not described by Family-Vicsek (9). Let us consider a dynamic scaling form for $S(k,t)$ as in Eq. (8) but with the scaling function $s_{\rm FV}(u)$ replaced by

$$s_A(u)\sim\begin{cases}u^{2\theta} & \text{if}\quad u\gg1\\ u^{2\chi+d} & \text{if}\quad u\ll1,\end{cases} \tag{12}$$

where the label A denotes intrinsic anomalous spectrum. Here θ is a *new* exponent which ''measures'' the anomaly in the spectrum. In a system of size L, Eqs. (8), (12) hold only up to the saturation time $t_s(L)=L^z$, after which the system size L replaces the correlation length $t^{1/z}$ in all expressions. In particular, at saturation the structure factor depends on $S(k,t)\sim L^{2\theta}k^{2\theta-(2\chi+d)}$. As a consequence, the stationary spectrum shows severe finite size effects, to the extent that it is not defined in the thermodynamic limit $L\rightarrow\infty$.

A scaling behavior such as Eqs. (8), (12) for the structure factor does not affect the behavior of the *global* width, which preserves its Family-Vicsek form, $W(L,t)\sim t^{\beta}$ for $t\ll L^z$ and $W(L,t\gg L^z)\sim L^{\chi}$. On the contrary, the *local* properties of the surface change dramatically if $S(k,t)$ scales as in Eq. (12). That can be seen by computing the height-difference correlation function from Eq. (7), which as before gives $G(\ell,t)\sim t^{2\beta}$ for times $t\ll\ell^z$. However, for intermediate times $\ell^z\ll t\ll L^z$ the integral (7) now picks up a nontrivial contribution from the behavior of $s_A(u)$ at large arguments, so that

$$G(\ell,t)\sim\ell^{2(\chi-\theta)}t^{2\theta/z}. \tag{13}$$

Thus the complete scaling of the height-difference correlation function (or, equivalently, the square of the local width) can be written as

$$G(\ell,t)\sim\left\{\begin{array}{lll} t^{2\beta} & \text{if} & t^{1/z}\ll\ell\ll L\\ \ell^{2\chi_{\rm loc}}t^{2\beta_*} & \text{if} & \ell\ll t^{1/z}\ll L\\ \ell^{2\chi_{\rm loc}}L^{2\theta} & \text{if} & \ell\ll L\ll t^{1/z}\end{array}\right\}=\ell^{2\chi}g_A(\ell/\xi(t)), \tag{14}$$

where $\xi(t)\sim t^{1/z}$ for $t^{1/z}\ll L$, and $\xi(t)=L$ for $t^{1/z}\gg L$. In Eq. (14), the local roughness and growth exponents are $\chi_{\rm loc}=\chi-\theta$, $\beta_*=\theta/z=\beta-\chi_{\rm loc}/z$, and the scaling function $g_A(u)$ is *not* constant for small arguments, but behaves as

$$g_A(u)\sim\begin{cases} u^{-2(\chi-\chi_{\rm loc})} & \text{if} \quad u\ll 1\\ u^{-2\chi} & \text{if} \quad u\gg 1.\end{cases} \tag{15}$$

Note that, to derive Eqs. (14), (15), no use is made concerning the value of χ below or above one.

The fact that $\theta\neq 0$ in Eq. (12) yields a local roughness exponent $\chi_{\rm loc}\neq\chi$ and an anomalous growth exponent $\beta_*\neq 0$. Therefore there are now *three* independent exponents describing the scaling properties of the surface, whereas for Family-Vicsek scaling (even in the presence of superroughening) there are only two. Contrary to the anomalous scaling due to superroughening, where no relevant physics exists on small scales, $\theta\neq 0$ implies *nontrivial* dynamics of the local fluctuations. The scaling behavior (14) of the local width is formally equivalent to that of superroughening (for that case $\chi_{\rm loc}=1$ and $\beta_*=\beta-1/z$), a fact which has produced some confusion in previous works where both anomalies have been identified in some way. Nevertheless, some discrete models with $\chi>1$ have been studied in the context of epitaxial growth [9] in which the observed anomalous scaling of $G(\ell,t)$ has indeed been associated with an anomalous structure factor of the form (8), (12). Specifically, in those references the following behavior has been proposed for the height-difference correlation function:

$$G(\ell,t)=\ell^{2\chi_{\rm loc}}t^{2\kappa}g(\ell/t^{(1-2\kappa)/z_S}), \tag{16}$$

where the scaling function behaves as $g(u)\sim$const. for $u\ll 1$ and $g(u)\sim u^{-2\chi}$ for $u\gg 1$. It has been argued in [9] that this scaling behavior should be expected whenever large slopes develop on the surface (thereby the tacit expectation of $\chi>1$). However, as seen above (and cf. Sec. III), Eq. (12) implies Eq. (14) even when $\chi<1$. Using the hyperscaling relations $z=2\chi+d$, $z_S=2\chi_{\rm loc}+d$ valid for the conserved relaxation rules of the models studied [9], it can be seen that $\kappa=\beta_*$, $z_S=z(1-2\beta_*)$ and formula (16) becomes identical to the above Eq. (14).

In view of the local scaling, Eq. (14), the structure factor can be conveniently rewritten as

$$S(k,t)\sim\begin{cases} t^{(2\chi+d)/z} & \text{if} \quad kt^{1/z}\ll 1\\ k^{-(2\chi_{\rm loc}+d)}t^{2(\chi-\chi_{\rm loc})/z} & \text{if} \quad kt^{1/z}\gg 1.\end{cases} \tag{17}$$

In this equation we can observe two interesting facts that characterize *intrinsic anomalous scaling*. First, $S(k\gg t^{-1/z},t)$ decays as $k^{-(2\chi_{\rm loc}+d)}$, and *not* following the $k^{-(2\chi+d)}$ law characteristic of Family-Vicsek scaling (superroughening case included). Second, there is an unconventional dependence of $S(k\gg t^{-1/z},t)$ on time which leads to nonstationarity of the structure factor. The combination of these two facts allows us to distinguish between anomalous scaling due to superroughening ($\chi>1$) and intrisic anomalous scaling.

III. NUMERICAL EXAMPLE: RANDOM DIFFUSION MODEL

In this section we present numerical simulations of a particular growth model in order to illustrate the theory we have just discussed in the preceding section. We study a random diffusion model in which one can tune the values of the global exponents χ, β. We compare the exact solution of the model with numerical integrations of the equation of motion. By analyzing the structure factor, we will see that it constitutes an excellent example of what we have termed intrinsic anomalous scaling in the preceding section, independent of the value of χ below or above one.

Let us consider the growth model in $1+1$ dimensions provided by the stochastic diffusion equation with a random diffusion coefficient

$$\frac{\partial h(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(D(x)\frac{\partial}{\partial x}h(x,t)\right)+\eta(x,t). \tag{18}$$

The diffusion coefficient $D(x)>0$ is distributed according to the probability density $P(D)=N_{\phi}D^{-\phi}f_c(D/D_c)$, where the parameter ϕ characterizes the intensity of the disorder. N_{ϕ} is merely a normalization constant and the cutoff function is $f_c(y)=1$ for $y\leqslant 1$ and $f_c(y)=0$ for $y>1$. If $\phi<0$, disorder in the diffusion coefficient does not play any role and the Edwards-Wilkinson [15] exponents $\chi=\chi_{\rm loc}=1/2$, $\beta=1/4$ are recovered. Thus the disorder is termed *weak*. On the contrary, for *strong* disorder, $0<\phi<1$, the critical exponents

χ,β are disorder dependent through the value of ϕ. From now on, we will restrict ourselves to this latter nontrivial case.

There are two major features that make model (18) interesting. First, it is linear and can be solved exactly [16,11]. As a consequence, exponents are dependent on the parameter ϕ in a known way. Specifically, in the case of interest here $(0<\phi<1)$ it was shown in [11] that

$$\beta=\frac{1}{2(2-\phi)},\quad \chi=\frac{1}{2(1-\phi)},\quad \chi_{\rm loc}=\frac{1}{2}. \tag{19}$$

As we see, the global roughness exponent χ can take a continuous range of values from $\chi=1/2$ to $\chi=\infty$ when varying the intensity of disorder from $\phi=0$ to 1. It is a curious property of model (18) that $\chi_{\rm loc}=1/2$ remains constant when changing the intensity of the disorder, in contrast to the wide range of variation existing in χ. This is a feature also found for many models of rough epitaxial growth [10].

The second feature that adds to the interest of the random diffusion model (18) is that, as shown in Ref. [11], it *always* exhibits anomalous scaling for $0<\phi<1$. Hence anomalous scaling occurs for surfaces with roughness exponent $\chi>1$ *as well as* for those with $\chi<1$. In particular, as seen above, the anomalous growth exponent β_* is given by the scaling relation $\beta_*=\beta-\chi_{\rm loc}/z$, and from Eq. (19) β_* depends on ϕ as $\beta_*=\phi/(4-2\phi)$.

Let us take as examples the cases of $\phi=2/3$ and $\phi=1/3$. In the former, the above formulas yield $\chi=3/2>1$ and anomalous local width with $\beta_*=1/4$. In the latter case $\chi=3/4<1$, however, one still gets anomalous scaling with $\beta_*=1/10\neq 0$. All these exponent values are consistent with a numerical integration of Eq. (18), see [11]. We have determined the scaling function $g_A(u)$ of the local width for these two values of $\phi=2/3,1/3$ through the data collapse shown in Fig. 1, in which the above exponents have been used. For both degrees of disorder $\phi=2/3$ and 1/3, the corresponding scaling functions we obtain are exactly of the form expected in the case of intrinsic anomalous scaling, Eq. (15), with $\chi_{\rm loc}=1/2$, $\chi=3/2$, and $\chi=3/4$, respectively.

Next we show that the anomalous scaling of the height-difference correlation for all values of $0<\phi<1$ in the present model is due to an intrinsically anomalous structure factor. We have calculated $S(k,t)$ in systems of sizes $L=16,\ldots,512$. Figure 2 shows our results for $L=128$, and 200 realizations of the disorder. For both values of the disorder parameter $\phi=2/3,1/3$ the spectrum decays as $k^{-(2\chi_{\rm loc}+1)}$ (not as $k^{-(2\chi+1)}$) and is clearly shifted for different times. This scaling behavior is the one in Eq. (17), which we associated in Sec. II with that of growth models having an intrinsic anomaly. This can be better appreciated when collapsing the curves of Fig. 2 as shown in Fig. 3, which displays the $S(k,t)$ data collapses for $\phi=2/3,1/3$, and yields a scaling function $s_A(u)$ with a form consistent with the intrinsic anomalous form Eq. (12), and *not* with the Family-Vicsek one, Eq. (9) [17]. As we see, model (18) clearly shows that intrinsic anomalous power spectra induce anomalous scaling of the height-difference correlation function (or equivalently the local width) both in the case of a global roughness exponent $\chi>1$ (e.g., for $\phi=2/3$) *and* in the case of $\chi<1$ (e.g., for $\phi=1/3$).

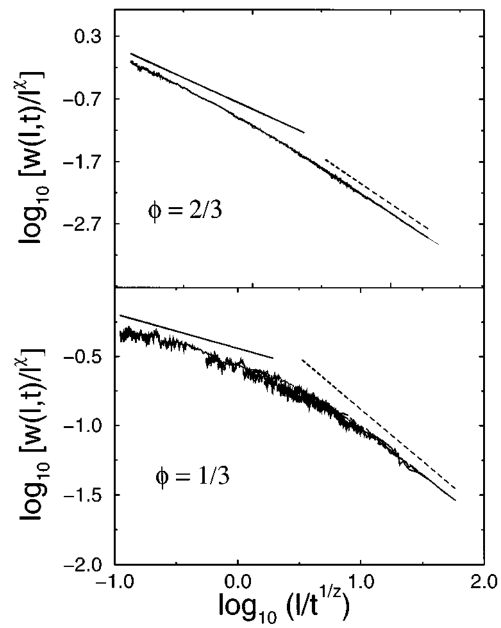

FIG. 1. Data collapse of the local width for the random diffusion model. In the upper panel ($\phi=2/3$), the exponents $\chi=3/2$, $z=4.08$ have been used. The straight lines are plotted as a guide to the eye having slopes $-3/2$ (dashed) and -1 (solid). For $\phi=1/3$ (lower panel), the data show a good collapse for $\chi=3/4$, $z=2.42$. The straight lines have slopes -0.25 (solid) and -0.75 (dashed). In both panels the scaling agrees with formula (15).

IV. CONCLUSIONS

The main conclusion of the present work is that anomalous scaling of rough surfaces is a more general (and independent) phenomenon than those cases associated with su-

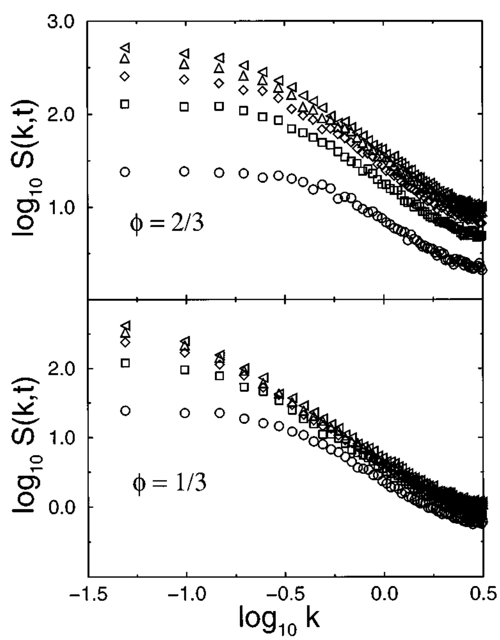

FIG. 2. Structure factor of the random diffusion model for two different degrees of the disorder and times $t=10^2$, 3×10^2, 5×10^2, 7×10^2, 9×10^2. Upper panel, results for $\phi=2/3$ ($\chi=3/2$). Lower panel, data for $\phi=1/3$ ($\chi=3/4$). The shift in time reflects the intrinsic anomalous character of the scaling.

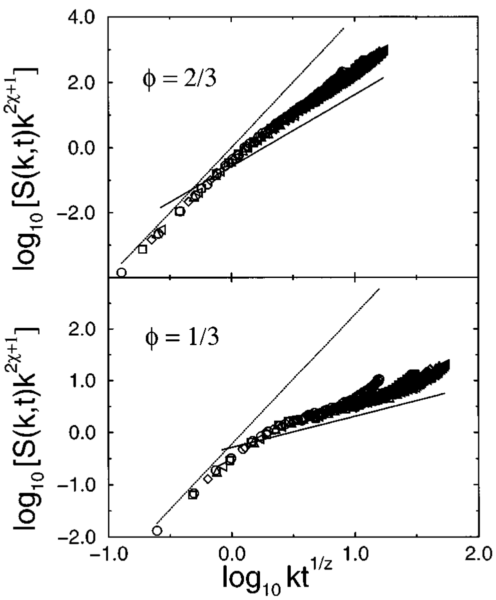

FIG. 3. Data collapse of the graphs in Fig. 2. The exponents used for the collapse in the upper panel ($\phi=2/3$) are $\chi=3/2$ and $z=4.08$. The straight lines have slopes 2.2 (solid) and 4 (dashed). In the lower panel ($\phi=1/3$) the exponents used for the collapse are $\chi=3/4$ and $z=2.42$. The straight lines have slopes 0.6 (solid) and 2.5 (dashed). In both panels the scaling function is described by Eq. (12).

perroughening. In this sense several issues associated with anomalus scaling remain to be clarified. One of them is its relation and/or interplay with the phenomena of multifractality and intermittency, known to take place in the superrough case [18,19]. Moreover, we have seen that the intrinsic anomalous surfaces display a novel type of spectrum thus far not considered for self-affine geometries, which are usually defined as having a Family-Vicsek spectrum [20]. An outstanding issue in this regard is to clarify the physical meaning of the exponent θ, as well as extending the currently available renormalization group techniques to be able to calculate the value of this exponent. On the phenomenological side, when performing experiments or interpreting numerical simulations of discrete growth models, we believe the existence of intrinsic anomalous scaling is a very relevant issue. On one hand, in the presence of intrinsic anomalous scaling the various correlation functions behave somewhat similarly to the Family-Vicsek case. However, there are important differences. For instance, $S(k,t)$ *scales* as $k^{-(2\chi_{\rm loc}+1)}$, and *not* as $k^{-(2\chi+1)}$, the graphs of $w(l,t)$ and $S(k,t)$ are shifted for different times, etc. There do exist data collapses, but again they are different from the usual Family-Vicsek type. For instance, if instead of plotting $w(\ell,t)/\ell^{\chi}$ vs $\ell/t^{1/z}$ as recommended in [10] and done here in Fig. 1 one chooses to represent $w(\ell,t)/t^{\beta}$ vs $\ell/t^{1/z}$ as frequently done in the literature, one gets, in the Family-Vicsek case,

$$\frac{w(\ell,t)}{t^{\beta}}\sim\begin{cases}\text{const} & \text{if} \quad \ell/t^{1/z}\gg 1\\ \left(\dfrac{\ell}{t^{1/z}}\right)^{\chi} & \text{if} \quad \ell/t^{1/z}\ll 1,\end{cases} \tag{20}$$

whereas for intrinsic anomalous scaling

$$\frac{w(\ell,t)}{t^{\beta}}\sim\begin{cases}\text{const} & \text{if} \quad \ell/t^{1/z}\gg 1\\ \left(\dfrac{\ell}{t^{1/z}}\right)^{\chi-\theta}=\left(\dfrac{\ell}{t^{1/z}}\right)^{\chi_{\rm loc}} & \text{if} \quad \ell/t^{1/z}\ll 1,\end{cases} \tag{21}$$

that is, in both cases there is data collapse with a very similar shape of the scaling function. However, the slopes of the corresponding scaling functions for large arguments are different. In the Family-Vicsek case, the slope coincides with the value of the exponent assumed to achieve the collapse of the data. However, in the anomalous case it does *not*. Therefore it is crucial to check whether the slope of the scaling function does or does not coincide with the assumed exponents. We believe this has not always been done when analyzing data from experiments and/or numerical simulations, and may have added to certain confusion existing in the literature in the identification of universality classes. Moreover, the intermediate time regime existing in surfaces with anomalous scaling introduces difficulties in the evaluation of exponents through the common use of local measurements such as the local width, frequently employed in experiments and numerical simulations. This may lead to the assignment of erroneous effective values (see a discussion in [11]) to the exponents of surfaces hypothesized to behave in the simple Family-Vicsek fashion, while a more accurate description of their true scaling properties may come through the use of the anomalous scaling ansatz.

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